ALLOWANCE FOR IONIZATION IN THE ANALYSIS OF A MULTICOMPONENT TURBULENT BOUNDARY LAYER

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Diffusion in a multicomponent turbulent boundary layer is investigated with allowance for ionization.

It is a well-known fact that flow in a multicomponent boundary layer at high temperatures is complicated by the presence of an electric field, which is elicited by the occurrence of electrically charged components: electrons and ions. Laminar boundary layers have been investigated with regard for ionization in several papers [1, 2].

We propose to analyze a partially ionized turbulent boundary layer. Earlier in [3] we set forth a method for the analysis of a chemically reactive multicomponent turbulent boundary layer of electrically neutral components. The advent of ionization introduces definite modifications into the equations, but the general scheme of the calculations for an ionized boundary layer on the assumption of quasi neutrality of the mixture and the absence of an electric current (similar assumptions were made in [1, 2]) is basically the same as for a boundary layer of neutral components. In the present note, therefore, we shall be mainly concerned with the differences engendered by the intorduction of ionization, omitting the reasoning and computations analogous to those set forth in [3] for a mixture of electrically neutral components.

We consider a turbulent boundary layer consisting of electrons (subscript 1), ions (subscript k, k = 2, ..., p), and neutral components, the number of which is N−p, where N is the total number of components. We investigate the "frozen" boundary layer, for which chemical reactions and recombination take place only on the surface of the flow obstacle. Inasmuch as the surface temperature of the obstacle in many cases is much lower than the temperature of the gas in the flow core, we are justified in assuming that the densities of ions and electrons at the obstacle wall are equal to zero. We also postulate quasi neutrality of the mixture [4] and the absence of electric current (the latter assumption is valid for gas flow past surfaces that are insulators). The conditions of quasi neutrality and absence of electric current are stated analytically as follows:

$$\sum_{1}^{p} \frac{c_i}{m_i} e_i = 0, \quad \sum_{1}^{p} \frac{J_i}{m_i} e_i = 0. \tag{1}$$

We consider a double-layer flow scheme, i.e., a turbulent core plus laminar substrate. The flow in the laminary substrate is described by the one-dimensional continuity, motion, diffusion, energy, and state equations in the same form as in [3].

The relation obtained from the Stefan–Maxwell equations between the diffusion fluxes and concentration grădients gains additional terms in the presence of charged components to account for the influences of the electric field on the diffusion process:

$$\frac{d}{dy}(c_i m) = \frac{m^2}{\rho} \sum_{j=1}^{N} \frac{1}{m_j D_{ij}} (c_i J_j - c_j J_i) + c_i m \frac{e_i E}{kT} \quad (i = 1, \ldots, N) \tag{2}$$

(for electrons $e_i = e$, for ions $e_i = ne$, and for the neutral components $e_i = 0$; we assume that the ionization multiplicity n is the same for all ions).

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Using conditions (1) and relation (2) for ions and assuming that the diffusion properties of the ions are identical and that the densities of ions and electrons are zero at the wall, we can show that the ion and electron density fields in the laminar substrate are similar, where the similarity is expressed by the relations

$$\frac{c_1}{I_1} = \frac{c_2}{I_2} = \ldots = \frac{c_p}{I_p}. \tag{3}$$

Here $I_i = J_i/(\rho v)_w + c_i = \text{const}$, and $(\rho v)_w$ is the mass flux at the wall.

The relations $I_i = \text{const}$ are the integrals of the diffusion equations for the laminar substrate [3].

From relations (2), written for ions ($i = 2, \ldots, p$) we eliminate the electric field, making use of relation (2) for electrons ($i = 1$); if we use Eqs. (3) in this operation, we reduce the Stefan–Maxwell relations for charged particles to the form

$$\frac{d(c_k m)}{dy} = \frac{m^2}{\rho}(\rho v)_w \frac{1}{n+1} \sum_{j=1}^{N} \left(\frac{1}{D_{kj}} + \frac{n}{D_{1j}}\right) \frac{1}{m_j}(c_k I_j - c_j I_k) \tag{4}$$
$$(k = 2, \ldots, p).$$

We introduce new variables by the formulas [5]

$$g_i = c_i m \ (i = 1, \ldots, N), \quad \xi = (\rho v)_w \int_0^y \frac{m dy}{\mu}.$$

Then the set of equations (4) reduces to a set of linear first-order equations with constant coefficients:

$$\frac{dg_k}{d\xi} = \sum_{j=1}^{N} \frac{S_{kj} + nS_{1j}}{1+n}(I_j g_k - I_k g_j) \quad (k = 2, \ldots, p). \tag{5}$$

For the neutral components the Stefan–Maxwell relations retain the same form in the new variables as in [3]:

$$\frac{dg_i}{d\xi} = \sum_{j=1}^{N} S_{ij}(I_j g_i - I_i g_j) \quad (i = p+1, \ldots, N). \tag{6}$$

We readily note on the basis of a comparison of Eqs. (5) and (6) that they differ in the appearance of the factor in front of the parentheses on the right-hand side of the equation. It may be inferred, therefore, that the diffusivity of the charged particles (ions) differs from that of the corresponding (i.e., having the same molecular weight) neutrals. Since the diffusivity of electrons is much greater than the diffusivities of ions, $S_{kj} \gg S_{1j}$; hence, we infer on the basis of Eqs. (5) and (6) that the diffusivities of the charged components (ions) are $(n + 1)$ times the diffusivities of the corresponding neutral components. A set of equations analogous to (5) and (6) has been solved in [3]. In the set of equations (5) we need merely solve one equation, because the density profiles of all other ions and electrons can be determined from relations (3).

The presence of ionization does not affect the solution of the motion and energy equations, and this solution is obtained by a procedure similar to that used in [3].

We now consider the turbulent part of the boundary layer. Because molar mixing prevails in the turbulent zone, it is reasonable to suppose that the diffusion characteristics of all the components are identical in that zone.

We write the analog of the Fick law for the ionized gas. From the Stefan–Maxwell relations we obtain for $D_{ij} = D_\varepsilon$

$$J_i = \rho D_\varepsilon \left(-\frac{\partial c_i}{\partial y} + \frac{E}{kT} c_i e_i\right).$$

Then the diffusion equation is represented as follows in the turbulent zone:

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{\partial}{\partial y} \rho D_\varepsilon \frac{\partial c_i}{\partial y} - \frac{\partial}{\partial y}\left(\rho D_\varepsilon \frac{E}{kT} c_i e_i\right). \tag{7}$$

If now we multiply every equation in the system (7) by $1/m_i$ and sum them on i from 1 to p, taking Eq. (1) into account, we obtain the equation

$$\rho u \frac{\partial X}{\partial x} + \rho v \frac{\partial X}{\partial y} = \frac{\partial}{\partial y} \rho D_\varepsilon \frac{\partial X}{\partial y}, \quad X = \sum_1^p \frac{c_i}{m_i}. \tag{8}$$

Assuming that the Prandtl number and all the Schmidt numbers formed from the turbulent transport coefficients are equal to unity in the turbulent part of the boundary layer, we deduce the following similarity between the fields of the velocity, enthalpy, densities of neutral components, and the quantity X [taking Eq. (8) into account]:

$$\frac{u - u^*}{u_\infty - u^*} = \frac{H - H^*}{H_\infty - H^*} = \frac{c_i - c_i^*}{c_{i\infty} - c_i^*} = \frac{X - X^*}{X_\infty - X^*}$$
$$(i = p+1, \ldots, N). \tag{9}$$

Here u is the velocity, H is the enthalpy, and the asterisk * and subscript ∞ refer to the values of the variables to the boundary of the laminary substrate and outer edge of the boundary layer, respectively.

It also follows from Eqs. (7) that the ion and electron density profiles in the turbulent part of the boundary layer are similar, this similarity being expressed by the formulas

$$\frac{c_1}{c_{1\infty}} = \frac{c_2}{c_{2\infty}} = \ldots = \frac{c_p}{c_{p\infty}}. \tag{10}$$

In order to effect closure of the problem we need to affix to the set of final relations (9) and (10) the equation of motion (as in [3], the equation of motion is conveniently written in integral form).

Equating the density values at the boundary of the laminar substrate on the turbulent side (10) and on the laminar side (3), we obtain the relations

$$\frac{c_{1\infty}}{I_1} = \ldots = \frac{c_{p\infty}}{I_p}. \tag{11}$$

Inasmuch as the densities $c_{i\infty}$ at the outer edge of the boundary layer are presumed to be known, we can use Eqs. (11) to express the values of $I_2, \ldots, I_p$ in terms of I_1.

Invoking the conditions at the boundary of the laminar substrate [3], including the equality of the velocities, enthalpies, and all particle densities on either side of the boundary of the laminar substrate, along with the conservation of momentum, mass of each component, and energy, we derive expressions for the values at the boundary of the laminar substrate. For example, the expression for the density of electrons at the boundary of the laminar substrate acquires the form

$$c_1^* = \frac{c_{1\infty} e^{\eta^*} + I_1 (B + 1 - e^{\eta^*})}{B+1}, \tag{12}$$

$$\eta^* = (\rho v)_w \int_0^y \frac{dy}{\mu}, \quad B = \frac{(\rho v)_w u_\infty}{\tau_w}.$$

Here τ_w is the wall friction.

Equation (12) coincides with the expression for the densities of neutrals at the boundary of the laminar substrate [3].

The densities of ions at the boundary of the laminar substrate can be determined from (3) and (12). The quantity I_1 is determined in the same way as I_i $(i > p)$ for neutral components [3].

The remainder of the calculations for an ionized turbulent boundary layer are analogous to the calculations for a multicomponent boundary layer of neutral components.

Thus, the most significant difference between the flows in a turbulent boundary layer of a partially ionized gas mixture and in a mixture of neutral gases is the increase induced by the electric field in the resultant binary diffusivities of the ions by comparison with the diffusivities of the neutral components in the laminar substrate.

NOTATION

x, y	are the coordinates;
u, v	are the velocity components;
c_i	is the mass density of the i-th component;
J_i	is the diffusion flux of the i-th component;
H	is the total heat content;
T	is the temperature;
ρ	is the density;
m	is the molecular weight of the mixture;
m_i	is the molecular weight of the i-th component;
μ	is the molecular viscosity;
D_{ij}	are the binary diffusivities;
D_ε	is the turbulent diffusivity;
E	is the electric field strength;
k	is the Boltzmann constant;
e_i	is the electric charge of the i-th component;
n	is the ionization multiplicity.

Subscripts

1	is the elements;
$2, \ldots, p$	is the ions;
$p + 1, \ldots, N$	is the neutral components;
∞, w, *	are the variables evaluated at the outer edge of the boundary layer, at the wall, and at the boundary of the laminar substrate, respectively.

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